(1)

Jr(H)- Maths paper-1st

W.Q.8. If S_n be the sum to n terms of the series

$\sin x + \sin 2x + \sin 3x + \dots$ to n terms

S.T. $\displaystyle\lim_{n\to\infty} \frac{S_1+S_2+S_3+\dots+S_n}{n} = \frac{1}{2}\cot\frac{x}{2}$

Here
$$S_n = \frac{\sin\frac{nx}{2}}{\sin\frac{x}{2}} \sin\left\{x+(n-1)\frac{x}{2}\right\}$$

$$= \frac{1}{2}\left\{\frac{2\sin\frac{nx}{2}\sin\left\{\frac{2x+nx-x}{2}\right\}}{\sin\frac{x}{2}}\right.$$

$$= \frac{1}{2\sin\frac{x}{2}}\left[2\sin\frac{nx}{2}\sin\left\{\frac{nx+x}{2}\right\}\right]$$

$$= \frac{1}{2\sin\frac{x}{2}}\left[\cos\left\{\cancel{\frac{nx}{2}}-\cancel{\frac{nx}{2}}-\frac{x}{2}\right\}-\cos\left\{\frac{nx}{2}+\frac{nx}{2}+\frac{x}{2}\right\}\right]$$

$$= \frac{1}{2\sin\frac{x}{2}}\left[\cos\left(-\frac{x}{2}\right)-\cos\left(nx+\frac{x}{2}\right)\right]$$

$$= \left[\frac{1}{2}\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{1}{2}\frac{\cos\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}\right]$$

$$\therefore S_n = \frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\frac{\cos\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$$

$$S_1 = \frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\frac{\cos\frac{3}{2}x}{\sin\frac{x}{2}}$$

$$S_2 = \frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\frac{\cos\frac{5}{2}x}{\sin\frac{x}{2}}$$

$$S_3 = \frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\frac{\cos\frac{7}{2}x}{\sin\frac{x}{2}}$$

$$- - - - - - - -$$

$$S_n = \frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\frac{\cos\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$$

$$\therefore S_1+S_2+\dots+S_n = \frac{n}{2}\cot\frac{x}{2} - \frac{1}{2\sin\frac{x}{2}}\left\{\cos\frac{3x}{2}+\cos\frac{5x}{2}+\dots+\cos\left(n+\frac{1}{2}\right)x\right]$$

$$= \frac{n}{2}\cot\frac{x}{2} - \frac{1}{2\sin x/2}\,\frac{\sin n.x/2}{\sin x/2}\cos\left\{\frac{3x}{2}+(n-1)\frac{x}{2}\right\}$$ ②

$$= \frac{n}{2}\cot x/2 - \frac{1}{2\sin^2 x/2}\,\sin\frac{nx}{2}\cos\frac{3x+nx-x}{2}$$

$$= \frac{n}{2}\cot\frac{x}{2} - \frac{1}{2\sin^2 x/2}\times \sin\frac{nx}{2}\cos\frac{nx+2x}{2}$$

$$\lim_{n\to\infty}\frac{s_1+s_2+\cdots+s_n}{n} = \lim_{n\to\infty}\frac{1}{2}\cot x/2 - \frac{1}{2\sin^2 x/2}\times\frac{\sin\frac{nx}{2}\cos\frac{(n+2)x}{2}}{n}$$

$$\therefore \lim_{n\to\infty}\frac{s_1+s_2+\cdots+s_n}{n} = \lim_{n\to\infty}\frac{1}{2}\cot x/2 - 0 \quad \text{Pr}$$

EX. 8

[problem Lot নাই]

1(i). please see w.o 1(i)

1(ii) $C = c\cos\alpha - \frac{1}{2}c^2\cos 2\alpha + \frac{c^3}{3}\cos 3\alpha - \cdots$ to ∞

$S = c\sin\alpha - \frac{1}{2}c^2\sin 2\alpha + \frac{c^3}{3}\sin 3\alpha - \cdots$ to ∞

$$\therefore C+iS = c(\cos\alpha+i\sin\alpha) - \frac{c^2}{2}(\cos 2\alpha+i\sin 2\alpha) + \frac{c^3}{3}(\cos 3\alpha+i\sin 3\alpha) - \cdots$$

$$= ce^{i\alpha} - \frac{c^2}{2}e^{i2\alpha} + \frac{c^3}{3}e^{i3\alpha} - \cdots$$

ধরি $x = ce^{i\alpha}$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$= \log(1+x) = \log(1+ce^{i\alpha})$$

$$= \log\{1+c(\cos\alpha+i\sin\alpha)\}$$

$$C+iS = \log\{(1+c\cos\alpha)+ic\sin\alpha\}$$

$$= \frac{1}{2}\log\{(1+c\cos\alpha)^2+c^2\sin^2\alpha\} + i\tan^{-1}\frac{c\sin\alpha}{1+c\cos\alpha}$$

$$\log(\alpha+i\beta) = \frac{1}{2}\log(\alpha^2+\beta^2) + i\tan^{-1}\beta/\alpha$$

or $C + iS = \frac{1}{2}\log\{1 + 2c\cos\alpha + c^2\cos^2\alpha + c^2\sin^2\alpha\} + i\tan^{-1}\frac{c\sin\alpha}{1 + c\cos\alpha}$

Equating real and imaginary parts, we get

$C = \frac{1}{2}\log\{1 + 2c\cos\alpha + c^2(\cos^2\alpha + \sin^2\alpha)\}$

$S = \tan^{-1}\frac{c\sin\alpha}{1 + c\cos\alpha}$ Ans

or $C = \frac{1}{2}\log\{1 + 2c\cos\alpha + c^2\}$ Ans